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TRAJECTORY SENSITIVITIES FOR SUN-MARS LIBRATION POINT MISSIONS

John P. Carrico[*], Jon D. Strizzi[‡], Joshua M. Kutrieb[‡], and Paul E. Damphousse[‡]

Previous research has analyzed proposed missions utilizing spacecraft in Lissajous orbits about each of the co-linear, near-Mars, Sun-Mars libration points to form a communication relay with Earth. This current effort focuses on 2016 Earth-Mars transfers to these mission orbits with their trajectory characteristics and sensitivities. This includes further analysis of using a mid-course correction as well as a braking maneuver at close approach to Mars to control Lissajous orbit insertion and the critical parameter of the phasing of the two-vehicle relay system, with one spacecraft each in orbit about L_1 and L_2. Stationkeeping sensitivities are investigated via a monte carlo technique. Commercial, desktop simulation and analysis tools are used to provide numerical data; and on-going, successful collaboration between military and industry researchers in a virtual environment is demonstrated. The resulting data should provide new information on these trajectory sensitivities to future researchers and mission planners.

INTRODUCTION

"NASA is seeking innovation to attack the diversity of Mars…to change the vantage point from which we explore…" - CNN, 25 June 2001

The concept of using communication relay vehicles in orbit about colinear Lagrange points to support exploration of the secondary body is not entirely new, being first conceptualized in the case of the Earth-Moon system by R. Farquhar REF. In addition, there have been many research efforts involving missions and trajectories to these regions.[6,9,11,14,19,20] A new approach on the concept that caught our interest was that introduced by H. Pernicka, et al, for a 2-satellite communications relay with one spacecraft in orbit about each of the co-linear, near Mars, Sun-Mars libration points, L1 and L2.[6] Further work by graduate researchers (Kok-Fai Tai and Danehy) refined this proposal and conducted investigations into the technical and fiscal aspects of such a mission, including trade studies on communication relay constellation options.[15,16] This analysis resulted in some favorable conclusions and rationale for a Mars communication relay system that utilizes 2-spacecraft in large amplitude Lissajous orbits, including system cost and performance measures comparable to a 3-spacecraft aerosynchronous system.

Recent research re-examined the 2-vehicle relay system orbiting the libration points, including transfer orbits, injection strategies, and stationkeeping, to see how past studies and data compared to that from current desktop computing techniques using full-

* Senior Astrodynamics Specialist, Analytical Graphics, Inc., Malvern, Pennsylvania
‡ Department of Aeronautics & Astronautics, US Naval Postgraduate School, Monterey, California

force model targeting and propagation (namely, the Satellite Tool Kit (STK) / *Astrogator* module).[17] Earth-Mars transfers and Lissajous orbit injections for a 2016 mission were analyzed. It was found that trajectory trends from the previous studies were still valid when using full force models, however the actual magnitudes of the maneuvers could increase. Also revealed was that using a braking maneuver at a low altitude (200 km) Mars periapsis prior to libration orbit insertion (LOI) saves significant spacecraft on-board fuel, for certain approach trajectories. One can take advantage of the geometry of this close approach to control the Z amplitude and class of the Lissajous orbit as well. It was also determined that the loose control technique for stationkeeping could be appropriate for the L1 and L2 communication relay concept; with anticipated annual stationkeeping costs similar to Sun-Earth orbits.

This report is an extension of this recent work, focusing on the key element of the 180 degree phasing of the two vehicle communication system, without which this concept will not work unless more vehicles are added to the system. This key parameter of the 180 degree phasing impacts the system in two ways: LOI such that the two vehicle system achieves the required phasing; and appropriate stationkeeping to maintain such a configuration. Transfer and injection trajectories that achieve the required phasing are presented, along with figures of merit to assess the communications relay coverage. Stationkeeping delta-v requirements are assessed via monte carlo simulation and analysis…

SUN-MARS LIBRATION POINT MISSIONS

The Sun-Mars libration points and the two-satellite relay system as originally proposed are shown in Figures 1 and 2, respectively. The concept places two satellites in orbit about the Sun-Mars L1 and L2 points to provide near continuous communications coverage for multiple vehicles on the surface of Mars and in orbit. This concept has many advantages over other options using relays in various orbits.[17]

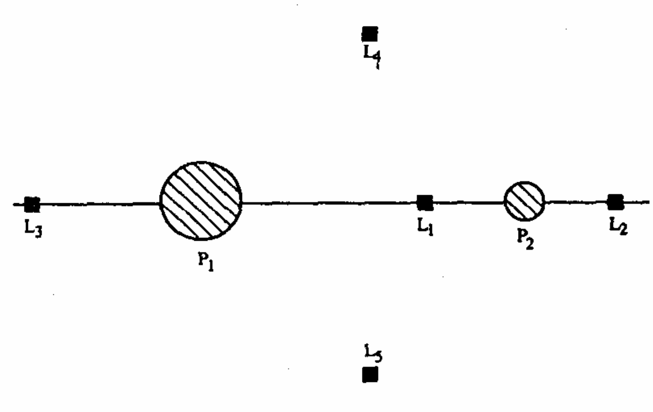


Figure 1 Geometry of the Lagrange Points of Two Primary Masses P_1 and P_2[7]

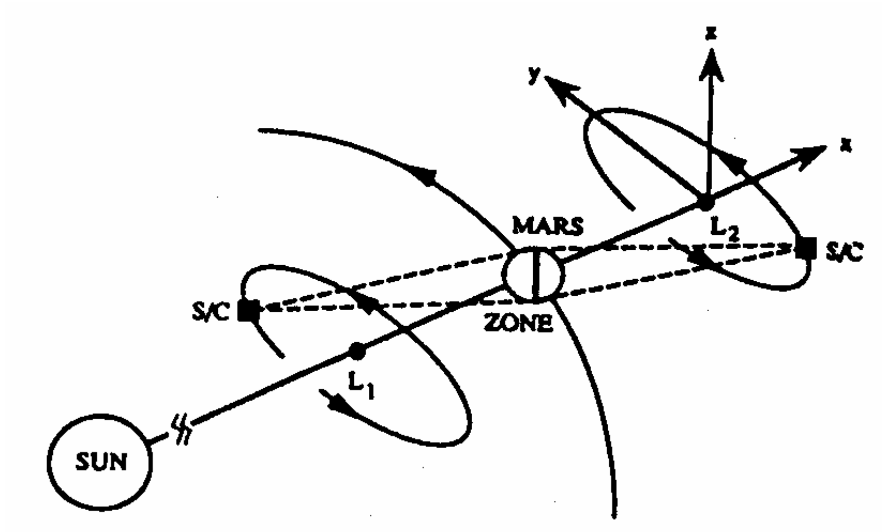


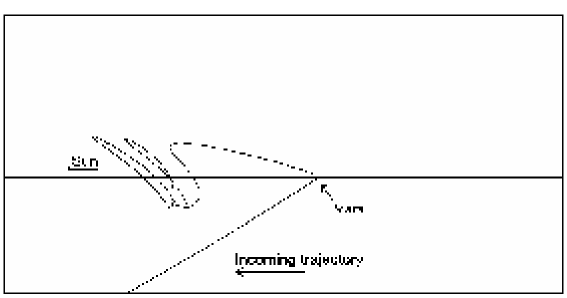
Figure 2 Sun-Mars L1 and L2 Halo Orbit Constellation[6]

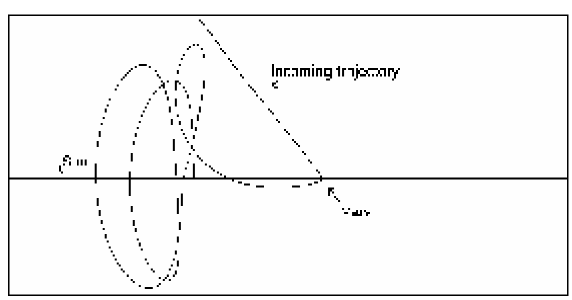
In designing the proper orbits in which to place the two satellites, the most important consideration is that they permit efficient maintenance of the 180 degree offset.[6] An additional consideration is that of avoiding having the satellite cross the "solar exclusion zone", the line between Mars and the Sun. Passing through this zone, communications would be disrupted due to intense solar interference. To avoid this problem the orbit must be large enough to avoid this crossing; an orbit of period greater than 0.9 years should suffice. Another obvious consideration is the choice of geometry and size of the orbit that reduces the required insertion maneuvers, and thus cost, from Earth.

MARS MISSION SIMULATIONS AND ANALYSIS

Earth-Mars Transfers and Libration Orbit Insertion (LOI)

Recent research investigated using a braking maneuver at close approach to Mars to lower the ΔV required for the LOI maneuver. [17] For the baseline 2003 transfer (Pern Ref) this data is reproduced below in Figures 3a (looking edge-on at the XZ plane) and 3b (looking down on the XY plane), and the table.



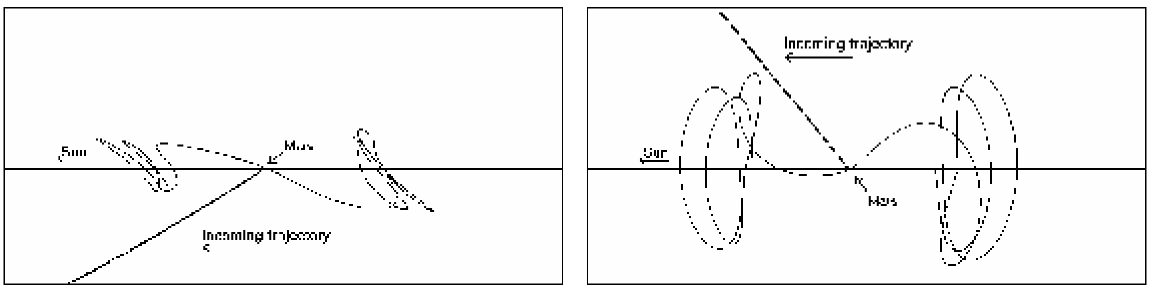


Figures 3a and 3b   L1 Orbit Insertion with Braking Maneuver

Table 1   Comparison of 2003 Transfers to L1 Orbit; 200 Day TOF

Scenario	**C3 Energy (km^2/sec^2)**	**Braking Δv (km/sec)**	**Orbit Insertion Δv (km/sec)**	**Total Δv (km/sec)**
Direct Injection	8.883	0	2.425	2.425
Braking Maneuver	9.056	0.856	0.104	0.960

The braking maneuver resulted in a ΔV savings of 1.465 km/sec, which would lead to fuel mass savings and/or increase in payload capacity. This type of maneuver seemed promising as a ΔV conserving technique, and was adopted for the 2016 mission simulations. A portion of that data is reproduced below.



Figures 4a and 4b   L1 and L2 Orbit Insertion with Braking Maneuver (2016)

Table 2 Comparison of 2016 Transfers to L1 Orbit for Different TOF

TOF (days)	C3 Energy (km^2/sec^2)	Braking Δv (km/sec)	Orbit Insertion Δv (km/sec)	Total Δv (km/sec)
181	8.847	2.314	0.047	2.360
200	10.377	1.710	0.047	1.757

Table 3 2016 Transfers to L1 & L2 Orbits for 200 Day TOF

Orbit	C3 Energy (km^2/sec^2)	Mid-course Δv (km/sec)	Braking Δv (km/sec)	Orbit Insertion Δv (km/sec)	Total Δv (km/sec)
L1	10.377	0	1.710	0.047	1.757
L2	10.377	0.001	1.708	0.085	1.795

Table 2 shows that a shorter TOF to Mars can be achieved with a lower C3 energy value, but that trajectory requires a larger braking maneuver than the longer transfer, to achieve the same mission orbit. This indicates that with these types of missions the lower energy transfer may not yield a lower braking and insertion ΔV specification.

Table 3 shows how two vehicles could start on the same transfer trajectory initially (as with a simultaneous launch) and the L2 vehicle targeted for it's close approach via a small mid-course correction. The simulation method is explained further in the section on targeting. The *total* TOF to Lissajous orbit insertion is different for each vehicle, which would assist in the phasing of the vehicles that is required for the communications relay system to maintain adequate coverage of Mars.

The two-vehicle simulation above does not attempt to achieve the proper phasing for actual relay mission operations. Of course, each vehicle could be launched separately to achieve the proper phasing, but with very constrained launch windows. The redundant launch costs may also make that approach cost-prohibitive. This paper focuses on scenarios where both spacecraft are launched on the same launch vehicle. As a result, the relative phasing of the spacecraft in their Lissajous orbits is controlled by their onboard propulsion. Three possible methods are investigated: using a midcourse maneuver (as above), adjustment of TOF from periapsis Mars to LOI, and the use of a Martian phasing loop.

Relative Phasing Selection

As noted previously, the key to obtaining sufficient communication coverage is to achieve 180 degree phasing of the two vehicles in their orbits; in other words, one is north of the Mars orbit plane while the other is south, and one is leading Mars while the other trails. This 180 degree phasing can be achieved by causing both spacecraft to reach their respective LOI points at the same time. In the previous study, the "baseline scenario" shown in Figure 4b had an LOI time difference of 56 days.

Trajectories to Achieve Two-Vehicle Phasing

The first method investigated to control the relative phasing was to use the mid course maneuver 30 days after launch to change the time of arrival of the spacecraft at Mars. By adjusting the time of arrival, the time of insertion into the Lissajous orbit (the LOI maneuver) would also be changed.

The baseline scenario in Figure 4b shows that the trajectories arriving at Mars are not symmetric with respect to the Sun-Mars rotating coordinate system; the incoming trajectories arrive from the L1 side of Mars. A consequence of this is that the L1 spacecraft inserts before the L2 spacecraft reaches its insertion point. A first step in getting both spacecraft to arrive at their LOI points simultaneously was to adjust the L2 spacecraft trajectory so that it would arrive at the periapsis Mars point earlier then the L1 spacecraft. To do this, however, required a very large midcourse maneuver. In addition, the decreased time of flight caused the incoming velocity at Mars to increase, and changed the direction of the incoming asymptote, as shown in Figure 5. In that figure, the dashed line is the original baseline and the solid represents an earlier perapsis Mars. The change in the asymptote angle in turn caused the time of flight from periapsis Mars to LOI to increase, and the epoch of the LOI point did not vary the same as the change in periapsis Mars epoch. In fact, for the case examined, an earlier periapsis epoch resulted in a two day *delay* in LOI. In addition, the retrograde braking maneuver and the LOI delta-V costs increased, again because of the change of the transfer trajectories. Table 4 shows the comparison data and the increase in delta-V. This method proved unfeasible because of the large delta-V cost associated with moving the epoch of LOI even a few days, and the direction of that movement for this case.

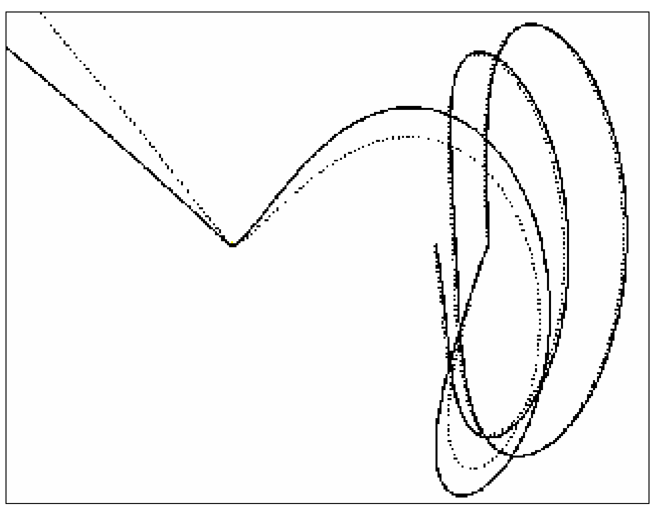
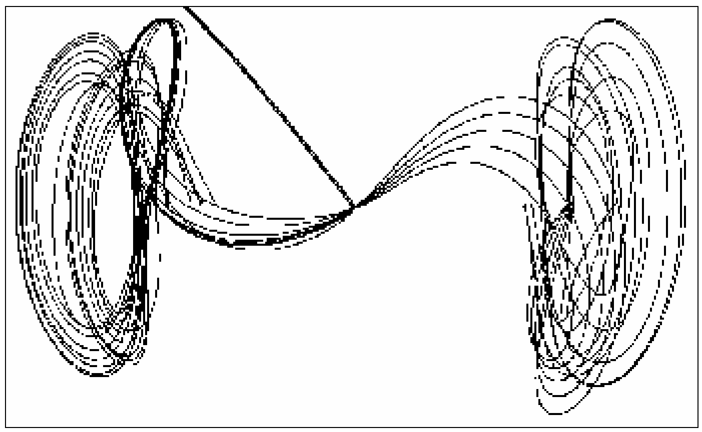


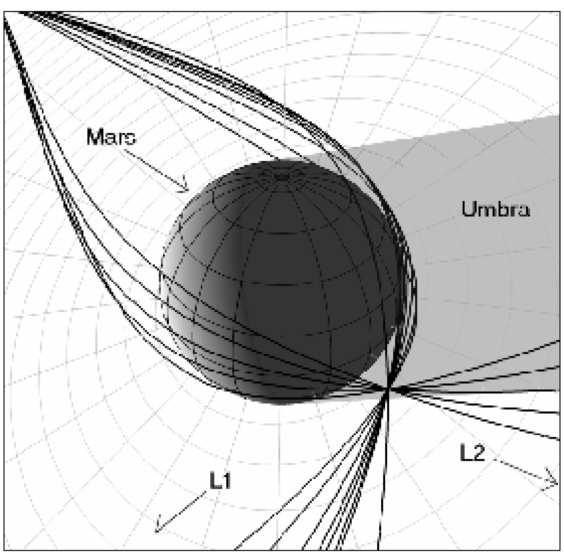
Figure 5: Mid-Course Maneuver Results in Earlier Periapsis Mars (solid line)

Table 4

The second method investigated to control phasing was to vary the time of flight of both vehicles from periapsis Mars to LOI. The time of flight is correlated with the amplitude, which is controlled by targeted B dot R value[17]. Thus, this TOF must be controlled to cause the L1 vehicle to insert later and have the L2 vehicle insert earlier so that their LOI times coincide. Our investigation results, detailed in the figures and Table 5 below, show that varying this TOF results in very large (and most likely unwanted) Lissajous orbit amplitudes (over 500,000 km) without achieving the synchronicity of the LOI required for the system phasing. However, there may be some applicability of this method to affect small changes in phasing if needed during operations.



Figure



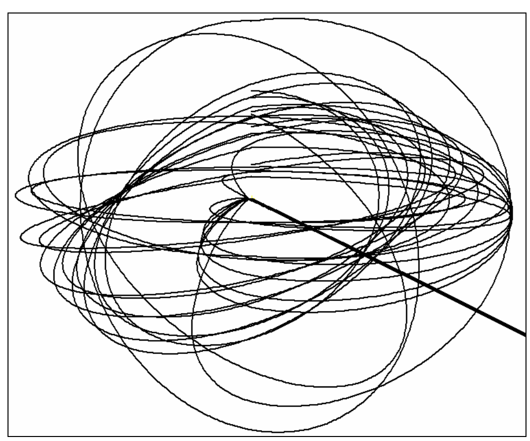


Figure 6a: Figure 6b: L1

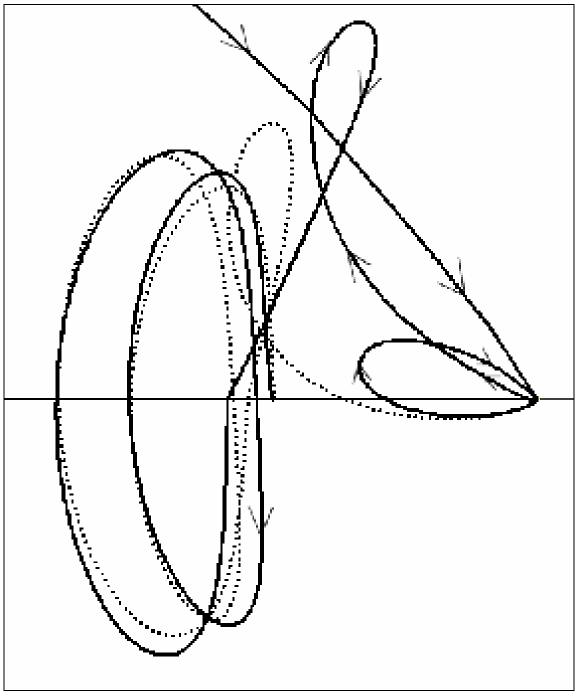
Table 5 – Z-axis amplitude and LOI date as a function of Time of Flight (TOF)

Transfer TOF (days)	Z-Amplitude at LOI (km)	Date of LOI	LOI diff from L1 Orig (Days)	Total Delta-V (km/s)
L1 Transfers				
184.39	-167,106.57	10 Mar 2017 21:27:05	0.00	1.7101691
188.77	-66,257.64	15 Mar 2017 06:26:07	4.37	1.7105497
194.64	68,229.80	21 Mar 2017 03:21:42	10.25	1.7106852
198.28	153,862.98	24 Mar 2017 18:43:20	13.89	1.7106055
199.98	195,445.37	26 Mar 2017 11:27:05	15.58	1.7105053
201.92	244,697.19	28 Mar 2017 10:11:20	17.53	1.7103805
203.05	273,937.27	29 Mar 2017 13:13:34	18.66	1.7102926
212.70	529,029.71	8 Apr 2017 04:46:19	28.31	1.7090500
L2 Transfers				
228.67	715,548.91	24 Apr 2017 04:07:05	44.28	1.7082738
230.62	542,729.85	26 Apr 2017 02:50:37	46.22	1.7090732
234.91	381,369.62	30 Apr 2017 09:50:30	50.52	1.7094752
240.82	235,000.62	6 May 2017 07:38:06	56.42	1.7097184
248.62	100,106.83	14 May 2017 02:49:45	64.22	1.7096993

Table 5 – Z-axis amplitude and LOI date as a function of Time of Flight (TOF)

The third method to control phasing is the use of a phasing loop orbit about Mars prior to LOI. With this approach, the L2 vehicle performs its LOI maneuver after Mars swingby and establishes the LOI time and phasing that the L1 vehicle must match. Using a retrograde capture maneuver at Mars periapsis, the L1 vehicle enters a phasing orbit about Mars. After one revolution in this orbit, a subsequent maneuver at periapsis transfers the vehicle out to the LOI point. The period of this phasing orbit summed with the time of flight of the transfer to LOI must be such that the desired epoch at LOI is achieved.

One might think that the phasing orbit period would be equal to the difference between LOI times for the L1 and L2 vehicles in the baseline configuration (56 days). However, since the phasing orbit periapsis point rotates in the Sun-Mars rotating frame, the transfer to LOI is longer than the baseline scenario. Thus, to obtain the correct total TOF the phasing orbit period is shorter than expected. In fact, specifying a phasing orbit simply equal to the 56 day time difference actually causes a time delay far in excess of that required. The data and figures below show these results. The use of a phasing orbit also introduces some flexibility into the execution of the entire mission. The result is that the 180 degree phasing of the two vehicles can be obtained.



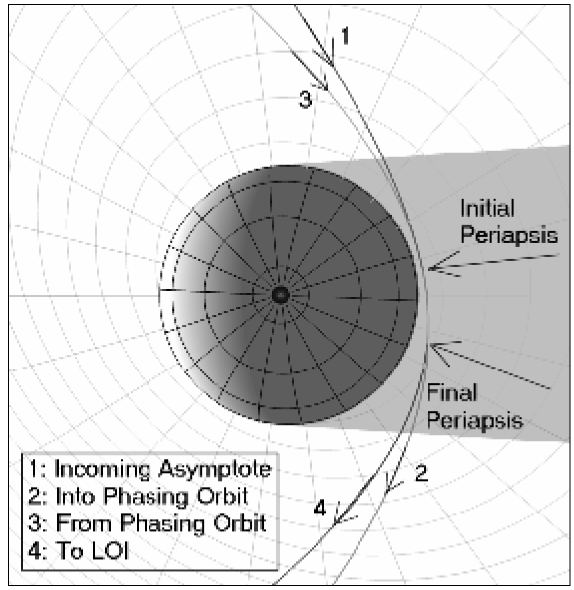


Figure 7a: Figure 7b:

Table 6

Estimation of Communication Coverage Achieved

Previous work has indicated that the 180 degree offset phasing of this communication system will allow near continuous coverage of the Martian surface so that exploration missions there would only experience communications loss for a few minutes twice a day, at the most. In order to verify and uncover more details on this, the effort, the authors analyzed the baseline 180 degree phased trajectories using the STK\Coverage module.

Two metrics were used to quantify the quality of coverage: "Maximum Revisit Time" and "Number of Gaps." Gaps are the times on the surface of Mars that are not within line-of-sight of either satellite. Maximum Revisit Time is defined as the maximum duration of the gap in coverage over the entire coverage interval, which starts at LOI and goes for 674 Earth days (over one Martian year). Number of gaps are the number of times in this same interval that contact with cannot be made with at least one of the satellites. Measured one longitude line.

Does take into account the rotation of Mars and its tilt, using fully integrated trajectories phased via the method in the proceeding section. As a special note, the south polar region had the longest revisit times (6 days), which occurred 4 times during the year. By inspection, it seems that there may be a seasonal variation that should be investigated. A complete investigation would include LOI epochs at various times of the Martian year and consider all locations on the planet.

Table XX

Latitude (deg)	Max Revisit Time (hrs)	Number of Gaps
90.0	0.000	1
80.0	3.639	61
70.0	5.117	335
60.0	3.499	584
50.0	1.707	902
40.0	0.696	890
30.0	0.494	846
20.0	0.441	785
10.0	0.463	733
0.0	0.486	688
-10.0	0.512	649
-20.0	0.543	609
-30.0	0.650	574
-40.0	0.845	524
-50.0	1.210	491
-60.0	5.994	436
-70.0	5.729	237
-80.0	7.425	130
-90.0	148.645	3

TARGETING METHODS USING STK/*ASTROGATOR*[1]

The simulations and analysis for this research are achieved with the same approach as in previous work. The trajectory targeting is done in a series of steps and phases, depending on the complexity and goals. They are summarized in the tables and explained below. The development of new targeting profiles was necessary to accomplish the objectives of the current research (achieving the required phasing). The purpose of the targeting is to determine the control variables necessary to achieve the particular transfers or maneuvers. The LOI maneuvers are similar to the energy balancing technique[16].

The transfer from the Earth to a Mars Lagrange orbit was targeted in a series of steps. The purpose of the targeting was to determine the control variables necessary to achieve this transfer. The initial orbit state represented the post launch Earth-centered hyperbolic trajectory. This was specified in target vector form, in the Earth-centered mean ecliptic and equinox of J2000 coordinate system. The seven parameters of the target vector are: epoch, radius of periapsis, C3 energy, right ascension (RA) and declination (Dec) of the outgoing hyperbolic asymptote, the velocity azimuth at periapsis, and the true anomaly. (Note: C3 is defined as negative the gravitational parameter of the central body divided by the semimajor axis. For hyperbolic orbits this is the square of the hyperbolic excess velocity.)

For this study, the epoch was chosen to match previous work, the true anomaly was set to zero, the velocity azimuth set to 90 degrees, and the radius of periapsis set to 6678.0 km. This represents a satellite near the Earth at perigee. The remaining parameters, C3 energy and the direction of the trajectory (RA and Dec of the asymptote) were used as control parameters. Two methods of insertion into Lagrange orbits were utilized and are discussed separately below.

Previous work described targeting directly into the Lissajous orbit. As mentioned earlier, the delta-V savigns are significant when using a close approach to Mars with a retrograde maneuver, and only this method is described.

Transfer using braking maneuver at Mars periapsis

This approach is shown in Table XX. The transfer to a Mars Lagrange point orbit using a braking maneuver at the close approach at Mars before the LOI maneuver was also targeted in stages. First, the target vector control parameters were adjusted by the differential corrector to achieve an epoch at periapsis Mars, and B-Plane components to place the trajectory on the anti-Sun side of Mars. Since this stage is just a first guess, the values used were B-dot-T of -10,000 km, and B-dot-R of 0.0 km.

The second step refined this to the desired close approach conditions. Using the same control parameters, the radius of close approach was used instead of B-dot-T, and was targeted to a radius of 3,600 km (about 200 kilometers altitude).

After the constraints at periapsis were met, the magnitude of a retrograde braking maneuver (anti-velocity direction) was used at periapsis to shape the trajectory until the trajectory crossed the XZ plane at the desired X distance in the Sun-Mars rotating libration-point coordinate system (X_{RLP}). After the retrograde maneuver was calculated, the LOI maneuver was planned using the same 4-step method previously described for the direct transfer.

The transfer to the L2 Lagrange orbit was planned in a similar manner, except that the trajectory must pass on the Sunward side of Mars at the close approach. This was done using a mid-course correction (MCC) maneuver as a control parameter, which also allowed the initial transfer parameters to be the same for both the L1 and L2 vehicles.

Table 6 Transfer and Insertion Using Braking Maneuver

Stage	Controls	Constraints	Dimension
I	C3 Targ.Vec. RA Targ.Vec. Dec	Periapsis Epoch B·T B·R	3x3
II	C3 Targ.Vec. RA Targ.Vec. Dec	Periapsis Epoch B·R $\lvert R_p \rvert$	3x3
III	ΔV_{retro}	1st XZ Plane Cross: X_{RLP}	1x1
IV - VII	Same as LOI	Same as LOI	3x3,1x1

Transfer to Achieve Relay Phasing

The first two methods attempted to control phasing used the approach described above. The 3rd method which utilized a phasing loop for the L1 satellite required a modified targeting procedure. The capture maneuver was targeted such that after one phasing loop and then subsequent transfer to LOI, the epoch of LOI would match the epoch of the L2 satellite LOI. In order to achieve this, two differential corrector targeting schemes were employed simultaneously. An inner targeter calculated the maneuver magnitude in the velocity direction needed to transfer from the phasing orbit to the Lissajous orbit. This inner targeter was "wrapped" by an outer targeter which calculated the retrograde maneuver at the first Mars periapsis. This targeter was set up to adjust the maneuver magnitude to achieve the epoch at LOI which occurs after the inner targeter has converged on a solution. Therefore, each time the outer targeter iterated and searched for the capture maneuver, the inner targeter was re-run to calculate the transfer from the phasing orbit to LOI.

STATIONKEEPING

Due to the precarious nature of the Lissajous orbit, precise and continuous stationkeeping (SK) techniques must be employed. Additionally, the precision required for certain missions located around the Sun-Mars Lagrange points requires the fidelity of such SK maneuvers to be extremely high. SK ΔVs as little as 1 mm/sec could be required.

Stationkeeping techniques fall into two major categories.[11] The first, referred to as a "tight" control technique, attempts to target the vehicle back to a nominal three-dimensional path. The second is the "loose" control technique that uses a simpler “orbital energy balancing” strategy to closely mirror a Lissajous orbit. The two control techniques differ only in the number of ΔV components that are varied. The loose technique will simply vary one component of ΔV while the tight technique varies two or more to achieve a nominal Lissajous orbit.

For future missions to Mars using the Sun-Mars Lagrange points, mission planners will have to consider several factors prior to making a decision on the SK technique to be used. Obviously mission requirements will dictate whether the loose control technique can be used to optimize SK costs or if the higher precision of the tight technique is necessary. In our example of a communication system in orbits about the L1 and L2 points, the loose technique should be sufficient as a communication system’s global nature does not depend on a precise halo orbit. Furthermore, the success of ACE and SOHO with using the loose technique has served to prove its utility and make it a preferred approach for scheduled missions to Sun-Earth Lagrange points.

Dunham and Roberts have shown that small ΔV errors on order of 0.1 mm/sec for the Sun-Earth/Moon system cause noticeable deviation from the nominal after about three revolutions in the Lissajous orbit. Previous work applied the same error to the Sun-Mars L1 Lissajous orbit as shown in Figure 5. [17] This error caused noticeable deviations after only one and a half revolutions. However, because the period of the Mars Lissajous is about twice that of the Earth Lissajous, the deviations occur approximately after the same duration. This is an indicator that the stationkeeping requirements for the Mars Lissajous will be on the same order as seen for the Earth missions, in terms of fuel used per year.

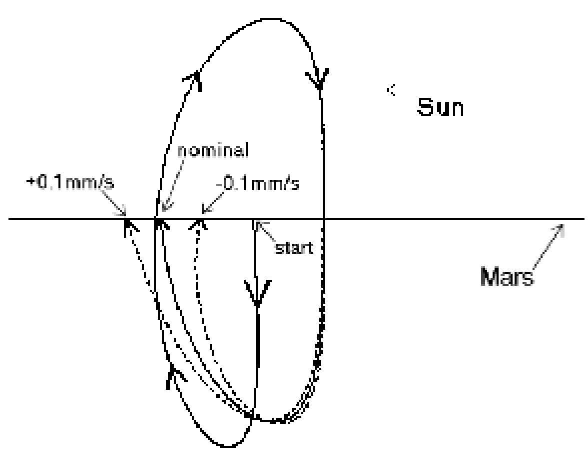


Figure 5 – Effect of small errors on Lissajous orbit

This current research effort explored the stationkeeping sensitivities of spacecraft in these orbits via Monte Carlo analysis. The uncertainties included those due to the orbit determination process, possible change in the effective area of the spacecraft affecting the Solar radiation pressure acceleration, and possible errors in the stationkeeping maneuver execution.

Monte Carlo Simulation Approach

The uncertainties were modeled as uncorrelated errors. The uncertainty magnitudes were: 100 meters in position, 10 cm/second in velocity, 10% uncertainty in the area of the spacecraft (could represent attitude changes), and a delta-v error of 10 cm/second (These could be attributed to both errors in execution of stationkeeping maneuver and attitude thruster control effects). A Monte Carlo simulation was setup to randomly vary these eight parameters and propagate the baseline L2 trajectory for 90 days. A stationkeeping maneuver was then targeted at that time to return the trajectory to that of a periodic orbit for the remainder of the Martian year. The statistics were gathered on the magnitude of the stationkeeping maneuver required to correct the trajectory.

The results of the Monte Carlo simulation with 100 runs yielded an average stationkeeping delta-v magnitude of 0.044 m/s (with standard deviation of 0.003). The same simulation was run for a large amplitude L2 orbit in the earth system (for comparison) and the average delta-v was 0.45 m/s and standard deviation of 0.03. Since the period of the Earth is approximately twice that of Mars, a second Earth centered L2 Monte Carlo run was made where the stationkeeping maneuver was done after only 45 days of propagation. The average delta-v was 0.43 m/s with the same standard deviation. These results seem to indicate that the Martian Lissajous orbits require an order of magnitude less stationkeeping delta-v than those in the Earth system. One possibility for this that was considered is Lunar effects; but the examination of a Lissajous about L2 of the Sun-Earth system with the moon removed yielded no significant difference in results. There are several other possibilities to be explored, including the different distances from the Sun and planet sizes.

CONCLUSION

Three methods were explored to achieve the 180 degree relative phasing of the spacecraft in their respective Lissajous orbits:

1. Adjusting the time of arrival at Mars periapsis using a midcourse correction;
2. Adjusting the time of flight from periapsis Mars to LOI by altering the amplitude of the Lissajous orbit; and
3. The addition of a phasing loop before the transfer to L1.

The first method proved too costly in terms of Delta-V. The second method did not move the LOI epochs close enough together. The third method was successful, and the targeting algorithm was described.

The quality of coverage was investigated using the fully numerically integrated trajectories and the actual motion of Mars' polar axis. For most latitudes, the maximum gap was found to be about a half an hour, which is slightly longer than previous papers suggested, but still within the scope of the missions described. The poles behave somewhat differently, with longer gaps, but far fewer. Future work could include investigation of the the effect of the phasing with the Martian seasons.

An estimate of the station keeping cost for a Mars L2 orbit was calculated using a Monte Carlo technique, varying the initial orbit state, area, and maneuver execution errors. This was compared with a similar Earth L2 orbit, and the Mars orbit requires about an order of magnitude less delta-V for the maneuver. The reasons behind this are not fully understood, and could be pursued in future work.

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